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The Spectrum of the Kinematic Dynamo Operator for an Ideally Conducting Fluid

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Abstract: The spectrum of the kinematic dynamo operator for an ideally conducting fluid and the spectrum of the corresponding group acting in the space of continuous divergence free vector fields on a compact Riemannian manifold are described. We prove that the spectrum of the kinematic dynamo operator is exactly one vertical strip whose boundaries can be determined in terms of the Lyapunov–Oseledets exponents with respect to all ergodic measures for the Eulerian flow. Also, we prove that the spectrum of the corresponding group is obtained from the spectrum of its generator by exponentiation. In particular, the growth bound for the group coincides with the spectral bound for the generator.

1. Introduction

In this paper we give a description of the spectrum of the kinematic dynamo operator and of the corresponding group it generates for an ideally conducting fluid in the space of continuous divergence free vector fields.

Consider a steady incompressible conducting fluid with Eulerian velocity $v = v(x)$ for $x \in \mathbb{R}^3$ and let ϕ^t denote the corresponding flow. The kinematic dynamo equations for the induction of a magnetic field $\mathbf{H}$ by the flow has the following form:

$$\dot{\mathbf{H}} = \nabla \times (v \times \mathbf{H}) + \varepsilon \Delta \mathbf{H}, \quad \operatorname{div} \mathbf{H} = 0\,, \tag{1.1}$$

where $\varepsilon = \mathscr{R}e_m^{-1}$, and $\mathscr{R}e_m$ is the magnetic Reynolds number (see, e.g., [15, Ch. 6]). The spectral properties of the kinematic dynamo operator L_ε, defined by (1.1), have been a subject of intensive study, in particular, in connection with the famous dynamo problem (see [1, 2, 3, 6, 16, 23] and references therein).

For the ideally conducting fluid, $\varepsilon = 0$, these equations become:

$$\dot{\mathbf{H}} = -(v, \nabla)\mathbf{H} + (\mathbf{H}, \nabla)v, \quad \mathbf{H}(x, 0) = \mathbf{H}_0(x), \quad \operatorname{div} \mathbf{H} = 0\,. \tag{1.2}$$

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