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Lie Algebra Cohomology and the Fusion Rules

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Abstract: We prove a vanishing theorem for Lie algebra cohomology which constitutes a loop group analogue of Kostant's Lie algebra version of the Borel–Weil–Bott theorem. Consider a complex semi-simple Lie algebra $\mathfrak{g}_{\mathbb{C}}$ and an integrable, irreducible, negative energy representation $\mathscr{H}$ of $L\mathfrak{g}_{\mathbb{C}}$. Given n distinct points z_k in $\mathbb{C}$, with a finite-dimensional irreducible representation V_k of $\mathfrak{g}_{\mathbb{C}}$ assigned to each, the Lie algebra $\mathfrak{g}_{\mathbb{C}[z]}$ of $\mathfrak{g}_{\mathbb{C}}$-valued polynomials acts on each V_k, via evaluation at z_k. Then, the relative Lie algebra cohomology $H^*\left(\mathfrak{g}_{\mathbb{C}[z]}, \mathfrak{g}_{\mathbb{C}}; \mathscr{H} \otimes V_1(z_1) \otimes \cdots \otimes V_n(z_n)\right)$ is concentrated in one degree. As an application, based on an idea of G. Segal's, we prove that a certain "homolorphic induction" map from representations of G to representations of LG at a given level takes the ordinary tensor product into the fusion product. This result had been conjectured by R. Bott.

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