Commun. Math. Phys. 173, 225 – 264 (1995)

Communications in Mathematical Physics

Analytic Bethe Ansatz for Fundamental Representations of Yangians

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Received: 30 June 1994 / in revised form: 4 November 1994

Abstract: We study the analytic Bethe ansatz in solvable vertex models associated with the Yangian $Y(X_r)$ or its quantum affine analogue $U_q(X_r^{(1)})$ for $X_r = B_r$, C_r and D_r. Eigenvalue formulas are proposed for the transfer matrices related to all the fundamental representations of $Y(X_r)$. Under the Bethe ansatz equation, we explicitly prove that they are pole-free, a crucial property in the ansatz. Conjectures are also given on higher representation cases by applying the T-system, the transfer matrix functional relations proposed recently. The eigenvalues are neatly described in terms of Yangian analogues of the semi-standard Young tableaux.

1. Introduction

1.1. General Remarks. Among many studies on solvable lattice models, the Bethe ansatz is one of the most successful and widely applied machineries. It was invented at the very dawn of the field [1] and is still providing rich insights. Meanwhile, Bethe's original idea has evolved into several versions of the Bethe ansätze called by the adjectives "thermodynamic" [2], "algebraic" [3], "analytic" [4, 5], "functional" [6] and so forth. These are all powerful techniques involving some profound aspects. We have yet to understand their full contents, a challenge raised on Feynman's "last blackboard" [7].

In this paper we step towards it by developing our recent works [8–11] further. We shall propose eigenvalue formulas for several transfer matrices in the models with the Yangian symmetry [12] or its quantum affine analogue [13–15]. An interesting interplay will thereby be exposed between the representation theory of these algebras and the analytic Bethe ansatz. Let us explain our basic setting of the problem.

1.2. Yang–Baxter Equation and Transfer Matrices. Consider the quantum affine algebra $U_q(X_r^{(1)})$ [13, 14] associated with any classical simple Lie algebra X_r of

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