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On the Classification of Diagonal Coset Modular Invariants

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Abstract: We relate in a novel way the modular matrices of GKO diagonal cosets without fixed points to those of WZNW tensor products. Using this we classify all modular invariant partition functions of $su(3)_k \oplus su(3)_1/su(3)_{k+1}$ for all positive integer level k, and $su(2)_k \oplus su(2)_l/su(2)_{k+l}$ for all k and infinitely many l (in fact, for each k a positive density of l). Of all these classifications, only that for $su(2)_k \oplus su(2)_1/su(2)_{k+1}$ had been known. Our lists include many new invariants.

1. Introduction

It is believed that a large subset of all rational conformal field theories can be generated from the Goddard–Kent–Olive (GKO) coset construction [11]. In the prototypical example, the minimal unitary series can be identified with the cosets $su(2)_k \oplus su(2)_1/su(2)_{k+1}$.

This paper is concerned with the classification of modular invariant partition functions for the diagonal GKO coset theories $g_k \oplus g_l/g_{k+l}$, where g_k is an untwisted affine algebra, at positive integer level k, with horizontal subalgebra g. We classify what are known as *physical invariants*: those modular invariants with non-negative integer multiplicities, and a unique vacuum; no further conditions are imposed. The connection between this problem and the WZNW one of finding partition functions for $g_k \oplus g_l \oplus g^c_{k+l}$ (where g^c_{k+l} is the dual of g_{k+l}) is well known, as is the method of constructing some of the partition functions for the coset by tensoring together partition functions for g_k, g_l, and g_{k+l}. But by means of a simple trick the coset classification is shown in Sect. 2 to be equivalent to a small subset of the classification for $g_k \oplus g_l \oplus g_{k+l}$ (this is more convenient to work with than $g_k \oplus g_l \oplus g^c_{k+l}$ – *e.g.* for finding exceptionals), that can be very easily identified (see Eq. (2.8c) below). In Sect. 3 we apply this to classify the coset physical invariants for certain levels k, l and $g = su(2)$ – half of these partition functions are not listed in *e.g.* [21, 4].

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