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A Microscopic Derivation of the Critical Magnetic Field in a Superconductor

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Abstract: The propagator for a noninteracting many electron system in a constant magnetic field in three space time dimensions is computed. This formula and the results of [FT1, 2] are used to give a microscopic derivation of a BCS-equation with magnetic field. It is shown that this equation has no solution if the magnetic field is sufficiently large. Perturbation theory in the interaction around the magnetic field propagator is discussed.

I. Introduction

In this paper, we consider the model of a many electron system in a constant magnetic field in three space time dimensions described by the effective potential

$$\mathcal{G}(\psi^e, \bar{\psi}^e) = \log \frac{1}{Z} \int e^{-\lambda \mathcal{V}(\psi+\psi^e, \bar{\psi}+\bar{\psi}^e)} d\mu_S \,, \tag{I.1}$$

where the interaction

$$\mathcal{V}(\psi, \bar{\psi}) = \sum_{\alpha,\beta \in \{\uparrow,\downarrow\}} \int d\xi d\xi' \bar{\psi}_\alpha(\xi) \psi_\alpha(\xi) V(\xi - \xi') \bar{\psi}_\beta(\xi') \psi_\beta(\xi') \tag{I.2}$$

is assumed to be short range and rotation invariant. Here, $d\mu_S$ is the Grassmann Gaussian measure with covariance S, where S is the exact propagator for a free many electron system in a constant magnetic field,

$$\begin{aligned} S(\mathbf{x}, \tau; \mathbf{x}'\tau') &= e^{i\frac{B}{2}(yx'-xy')} \frac{B}{2\pi} \sum_{n=0}^{\infty} l_n \left(\frac{B|\mathbf{x}-\mathbf{x}'|^2}{2} \right) \\ &\quad \times e^{-\varepsilon_n(\tau-\tau')} [\theta(-\varepsilon_n)\theta(\tau'-\tau) - \theta(\varepsilon_n)\theta(\tau-\tau')] \\ &= e^{i\frac{B}{2}(yx'-xy')} D(\mathbf{x}-\mathbf{x}', \tau-\tau') \,, \end{aligned} \tag{I.3}$$

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