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On Equivalence of Floer's and Quantum Cohomology

Vladimir Sadov

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA and L.D. Landau Institute for Theoretical Physics, Moscow, Russia

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Abstract: We show that the Floer cohomology and quantum cohomology rings of the almost Kähler manifold M, both defined over the Novikov ring of the loop space $\mathscr{L}M$, are isomorphic. We do it using a BRST trivial deformation of the topological A-model. The relevant aspect of noncompactness of the moduli of pseudoholomorphic instantons is discussed. It is shown nonperturbatively that any BRST trivial deformation of A model which does not change the dimensions of BRST cohomology does not change the topological correlation functions either.

1. Introduction

The "quantum cohomology" ring H_Q^* $(=(c,c)$ ring in terms of $N = 2$ sigma models) was introduced in [1], see also [2–6]. The infinite volume limit of H_Q^* coincides with the ordinary cohomology ring $H^*(M)$ of the target space M. For any finite volume, H_Q^* is a deformation of $H^*(M)$. A natural question arises about the meaning of this deformation in classical geometry.

One way to do this in terms of the moduli space of holomorphic instantons was introduced in [2, 6, 5]. It is more or less standard by now and we refer the reader to [5], for a review of that approach. Closely related to, but not quite the same as the latter one, is the interpretation in terms of geometry of the parameterized loop space $\mathscr{L}M$ of the target space, conjectured in [1, 3]. It turns out that an appropriate object to deal with in this context is what the mathematicians call a Floer symplectic cohomology H_F^* [7–9].

H_F^* appear via the Witten–Floer [10, 11, 7] complex in $\mathscr{L}M$, whose vertices are the fixed points of some symplectomorphism ϕ of M and the edges are the "pseudoholomorphic instantons" (defined below) connecting these fixed points. It is graded by the same abelian group 2Γ as the quantum cohomology H_Q^* (and for the same reason), a phenomenon known to physicists as the anomalous conservation of fermionic number. Under some natural assumptions [7, 12] one has $\dim H_F^\iota = \sum_{\gamma\in\Gamma} b^{\iota+2\gamma}(M)$, where on the left-hand side we identify the index of Betty numbers modulo 2Γ. Moreover, there is a natural action of $H^*(M)$ on H_F^*. It is