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Determinants of Dirac Boundary Value Problems over Odd-Dimensional Manifolds

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Abstract: We present a canonical construction of the determinant of an elliptic self-adjoint boundary value problem for the Dirac operator D over an odd-dimensional manifold. For 1-dimensional manifolds we prove that this coincides with the ζ-function determinant. This is based on a result that elliptic self-adjoint boundary conditions for D are parameterized by a preferred class of unitary isomorphisms between the spaces of boundary chiral spinor fields. With respect to a decomposition $S^1 = X^0 \cup X^1$, we explain how the determinant of a Dirac-type operator over S^1 is related to the determinants of the corresponding boundary value problems over X^0 and X^1.

1. Introduction

Let X be a compact odd-dimensional Riemmanian spin manifold with boundary Y. We assume there is a collar neighbourhood $U = [0,1] \times Y$ of the boundary in which the Riemannian metric is a product metric. Fix a choice of spin structure, and let S be the complex spinor bundle over X. The Dirac operator $D : C^\infty(X;S) \to C^\infty(X;S)$ is the first-order elliptic differential operator defined at $x \in X$ by $Ds = \sum_i e_i \cdot \nabla_{e_i} s$, where ∇ is the canonical metric connection on S and $\{e_i\}$ is an orthonormal frame for $T_x X$. The e_i act on S by Clifford multiplication. The restriction of S to Y may be identified with the spinor bundle over Y with Z_2 grading $S_Y = S^+ \oplus S^-$. That induces a decomposition of the boundary spinor fields $F = F^+ \oplus F^-$ into positive and negative chirality with respect to which the Dirac operator D_Y over the boundary splits into the chiral operator $D_Y^+ : F^+ \to F^-$, whose index is calculated by evaluating the $\hat{A}$-cohomology class over Y, and its formal adjoint D_Y^-. We assume that D_Y is invertible.

By a *boundary value problem* D_W for D, we shall mean D with restricted domain $C_W^\infty(X;S) = \{\psi \in C^\infty(X;S) : P_W b\psi = 0\}$, where $P_W : C^\infty(Y;S) \to C^\infty(Y;S)$ is a pseudodifferential projection operator (of order 0) with range W, and $b : C^\infty(X;S)$

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