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The Spectral Problem for the q-Knizhnik–Zamolodchikov Equation and Continuous q-Jacobi Polynomials

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Abstract: The spectral problem for the q-Knizhnik–Zamolodchikov equations for $U_q(\widehat{sl_2})(0 < q < 1)$ at arbitrary non-negative level k is considered. The case of two-point functions in the fundamental representation is studied in detail. The scattering states are given explicitly in terms of continuous q-Jacobi polynomials, and the S-matrix is derived from their asymptotic behavior. The level zero S-matrix is closely connected with the kink-antikink S-matrix for the spin-$\frac{1}{2}$ XXZ antiferromagnet. An interpretation of the latter in terms of scattering on (quantum) symmetric spaces is discussed. In the limit of infinite level we observe connections with harmonic analysis on p-adic groups with the prime p given by $p = q^{-2}$.

1. Introduction

There is accumulating evidence for the idea [1] that excitation scattering in integrable models is "geometric," i.e., that the corresponding wave functions are spherical functions of certain quantum symmetric spaces. With this idea in mind, we have recently derived [2] the physical S-matrix for the scattering of kinks and antikinks for the (spin-$\frac{1}{2}$) XXX and XXZ anti-ferromagnets, starting from the level zero q-Knizhnik–Zamolodchikov (q-KZ) equations for $U_q(\widehat{sl_2})$ in the fundamental representation (the Heisenberg XXX case corresponds to $q = 1$). These q-KZ equations [3] are q-deformations of the ordinary first order differential KZ equations [4], which, in turn, are similar to the familiar Dirac and Bargmann–Wigner [5] equations. That the q-KZ equations apply in the kink-antikink problem has to do with the fact that kinks and antikinks are known to be spin $\frac{1}{2}$ excitations [6, 7]. Here we make this connection more precise and extend this work to non-negative values of the level k. This brings the continuous q-Jacobi polynomials into play and physics-wise concerns $SL_q(k+2)$-magnetics (or the corresponding generalizations of Baxter's eight vertex model). The first-order matrix q-KZ operator is cast here

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