Commun. Math. Phys. 172, 551 – 569 (1995)

Communications in Mathematical Physics

The Analysis of the Widom-Rowlinson Model by Stochastic Geometric Methods

J.T. Chayes[1], L. Chayes[1], R. Kotecký[2]

[1] Department of Mathematics, UCLA, Los Angeles, CA 90024, USA. Email: jchayes@math.ucla.edu; lchayes@math.ucla.edu

[2] Center for Theoretical Study, Charles University, Ovocný trh 3, 116 36 Praha 1, Czech Republic E-mail: kotecky@cspuni12.bitnet

Received: 7 July 1994

Abstract: We study the continuum Widom-Rowlinson model of interpenetrating spheres. Using a new geometric representation for this system, we provide a simple percolation-based proof of the phase transition. We also use this representation to formulate the problem, and prove the existence of an interfacial tension between coexisting phases. Finally, we ascribe geometric (i.e. probabilistic) significance to the correlation functions which allows us to prove the existence of a sharp correlation length in the single-phase regime.

1. Introduction

1A. Background and statement of results. The Widom-Rowlinson model [WR] is a simple and beautiful model of continuum particles. It is of interest both because of it applicability in the description of continuum systems, and because it is the only continuum system for which a phase transition has been rigorously established [R]. The Widom-Rowlinson (WR) model has two equivalent standard formulations – one as a binary gas and the other as a single-species model of a dense (liquid) phase in contact with a rarefied (gas) phase. In the binary gas formulation, the only interaction is a hard-core exclusion between the two species of particles – call them A and B. There is no intraspecies interaction: two particles of the same type can interpenetrate freely. The phase diagram of the model is a function of the fugacities, z_A and z_B, of the two species. Clearly, there is a symmetry between A and B particles; hence $z_A = z_B = z$ is a line of symmetry of the phase diagram. For both the continuum and lattice versions of the model, it has been shown via Peierls' arguments that for z large enough, the symmetry is spontaneously broken, yielding two phases: one is A-rich and the other is B-rich [LG, R]. The transition between these phases is first-order. It is expected, but not proved that the line $z_A = z_B = z$ of first-order transitions ends in a critical point at some positive value $z = z_c$ of the common fugacity. The single-species formulation of the model is obtained by

Partly supported by the grants GAČR 202/93/0499, GAUK 376, NSF-DMS 91-04487, and NSF-DMS 93-02023