Commun. Math. Phys. 172, 187 – 220 (1995)

Communications in Mathematical Physics

The Quantum Structure of Spacetime at the Planck Scale and Quantum Fields

Sergio Doplicher[1], Klaus Fredenhagen[2], John E. Roberts[3]

[1] Dipartimento di Matematica, Università di Roma "La Sapienza", I-00185 Roma, Italy
[2] II Institut für Theoretische Physik der Universität Hamburg, D-22761 Hamburg, Germany
[3] Dipartimento di Matematica, Università di Roma "Tor Vergata", I-00133 Roma, Italy.
[1,3] Research supported by MRST and CNR-GNAFA

Received: 22 June 1994

Abstract: We propose uncertainty relations for the different coordinates of spacetime events, motivated by Heisenberg's principle and by Einstein's theory of classical gravity. A model of Quantum Spacetime is then discussed where the commutation relations exactly implement our uncertainty relations.

We outline the definition of free fields and interactions over QST and take the first steps to adapting the usual perturbation theory. The quantum nature of the underlying spacetime replaces a local interaction by a specific nonlocal effective interaction in the ordinary Minkowski space. A detailed study of interacting QFT and of the smoothing of ultraviolet divergences is deferred to a subsequent paper.

In the classical limit where the Planck length goes to zero, our Quantum Spacetime reduces to the ordinary Minkowski space times a two component space whose components are homeomorphic to the tangent bundle TS^2 of the 2-sphere. The relations with Connes' theory of the standard model will be studied elsewhere.

1. Introduction

It is generally believed that the picture of spacetime as a manifold M locally modelled on the flat Minkowski space $M_0 = \mathbb{R}^4$ should break down at very short distances of the order of the Planck length

$$\lambda_P = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 1.6 \times 10^{-33} \text{ cm} .$$

Limitations in the possible accuracy of localization of spacetime events should in fact be a feature of a Quantum Theory incorporating gravitation.

There have been investigations on possible mechanisms leading to such limitations in the context of string theory [1, 2], in Ashtekar's approach to quantum gravity [3], and, in a more formal way, in the context of Quantum Groups [4, 5]. These different approaches have led to different limitations and, more significantly, to different pictures of spacetime where gravitational effects in the small are necessarily strong (spacetime foam [6]).