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The Short Distance Behavior of $(\phi^4)_3$

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Abstract: We consider the ϕ_3^4 quantum field theory on a torus and study the short distance behavior. We reproduce the standard result that the singularities can be removed by a simple mass renormalization. For the resulting model we give an L_p bound on the short distance regularity of the correlation functions. To obtain these results we develop a systematic treatment of the generating functional for correlations using a renormalization group method incorporating background fields.

1. Introduction

The renormalization group is not a group, but a technique for isolating the singularities of a quantum field theory. Originally invented by Wilson it has become one of the standard tools used in rigorous work on the subject. Still, its application is far from routine.

In a series of papers starting with a paper by Brydges and Yau [BY90], the authors have developed a systematic version of the technique which we believe has substantial advantages [Bry92, DH91, DH92b, DH92a, DH93]. Until now the Brydges-Yau method has not been applied to ϕ^4 type models, but we have developed a modification (incorporating background fields) which covers this case as well. In this paper we use it to study the short distance problem for the ϕ_3^4 model. We believe it can be used for many other problems. The paper [BDH93] also reviews the general framework of the background field method.

Here is a brief history of rigorous work on the ϕ_3^4 model. The original stability estimate was given by Glimm and Jaffe [GJ73] in a very difficult proof using a phase-cell cluster expansion. The complete construction of the model was finished by Feldman and Osterwalder [FO76] and Magnen and Sénéor [MS77]. Since then it has been worked over by many other authors, usually looking for a simpler proof. Some of the work continued to use a phase-cell cluster expansion, for example Battle and Federbush [BF83] and Williamson [Wil87]. Others used renormalization group

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