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Beta Function and Anomaly of the Fermi Surface for a d = 1 System of Interacting Fermions in a Periodic Potential

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Abstract: We derive a perturbation theory, based on the renormalization group, for the Fermi surface of a one dimensional system of fermions in a periodic potential interacting via a short range, spin independent potential. The infrared problem is studied by writing the Schwinger functions in terms of running couplings. Their flow is described by a Beta function, whose existence and analyticity as a function of the running couplings is proved. If the fermions are spinless we prove that the Beta function is vanishing and the renormalization flow is bounded for any small interaction. If the fermions are spinning the Beta function is not vanishing but, if the conduction band is not filled or half filled and the interaction is repulsive, it is possible again to control the flow proving the partial asymptotic freedom of the theory. This is done showing that the Beta function is partially vanishing using the exact solution of the Mattis model, which is the spin analogue of the Luttinger model. In both these cases Schwinger functions are anomalous so that the system is a "Lutttinger liquid." Our results extend the work in [B.G.P.S], where neither spin nor periodic potential were considered; an explicit proof of some technical results used but not explicitly proved there is also provided.

1. Introduction and Statement of the Results

We study by renormalization group techniques the analyticity properties of the Beta function and the behaviour of the pair Schwinger function for momenta near the Fermi surface for a one dimensional system of n fermions moving in a common periodic field $-\partial_{\vec{x}} U(\vec{x})$ and interacting by a short range pair potential. We consider both spinless $\sigma = 0$ or spinning fermions $\sigma = \pm 1/2$. The recent interest about interacting electrons in a periodic potential [D.M., Sh.] motivates our study. The one dimensional hamiltonian is

$$H = T + \lambda V \,, \tag{1}$$

$$T = \sum_{\sigma} \int_{-L/2}^{L/2} d\vec{x}\, \psi^{+}_{\vec{x},\sigma} \left(-\frac{\partial^2}{2m} + U(\vec{x}) - \mu \right) \psi^{-}_{\vec{x},\sigma} \,,$$