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Long-Time Effect of Relaxation for Hyperbolic Conservation Laws*

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Abstract: The hyperbolic conservation laws with relaxation appear in many physical systems such as nonequilibrium gas dynamics, flood flow with friction, viscoelasticity, magnetohydrodynamics, etc. This article studies the long-time effect of relaxation when the initial data is a perturbation of an equilibrium constant state. It is shown that in this case the long-time effect of relaxation is equivalent to a viscous effect, or in other words, the Chapman–Enskog expansion is valid. It is also shown that the corresponding solution tends to a diffusion wave time asymptotically. This diffusion wave carries an invariant mass. The convergence rate to this diffusion wave in the L^p-sense for $1 \leqq p \leqq \infty$ is also obtained and this rate is optimal.

1. Introduction

We consider the following model of hyperbolic conservation laws with relaxation [5, 8]:

$$u_t + f(u,v)_x = 0 \,, \tag{1.1}$$

$$v_t + g(u,v)_x = h(u,v), \quad -\infty < x < \infty, \quad t > 0 \,. \tag{1.2}$$

A concrete physical model is the following flood flow with friction:

$$h_t + (hu)_x = 0 \,,$$

$$(hu)_t + \left(hu^2 + \frac{gh^2}{2}\right)_x = (gh\tan\alpha - C_f u^2) \,,$$

where h is the height of the water, u, the velocity, g, the gravitation constant, α, the inclined angle of the river, and C_f, the friction coefficient of the river. Other physical models include the nonequilibrium gas dynamics, the viscoelasticity, the magnetohydrodynamics, etc. [8].

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