Commun. Math. Phys. 171, 661 – 682 (1995)

Communications in Mathematical Physics

Modifying the KP, the n^{th} Constrained KP Hierarchies and their Hamiltonian Structures

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Received: 22 March 1994/in revised form: 1 August 1994

Abstract: The Kadomtsev–Petviashvili (KP) hierarchy has infinitely many Hamiltonian pairs, the n^{th} pair of them is associated with L^n, where L is the pseudodifferential operator (PDO) [3, 4]. In this paper, by the factorization $L^n = L_n \cdots L_1$ with L_j, $j = 1, \ldots, n$ being the independent PDOs, we construct the Miura transformation for the KP, which leads to a decomposition of the second Hamiltonian structure in the n^{th} pair to a direct sum. Each term in the sum is the second structure in the initial pair associated with L_j. When we impose a constraint (1.9) (i.e. a new type of reduction) to the KP hierarchy, we obtain the similar results for the constrained KP hierarchy. In particular the second Hamiltonian structure for this hierarchy is transformed to a vastly simpler one.

1. Introduction

It has been known that the n^{th} Korteweg–de Vries (KdV) type (also called the Gelfand–Dickey) hierarchy associated with a scalar n^{th} order differential operator

$$L = \partial^n + u_{n-1}\partial^{n-1} + \cdots + u_0, \quad \partial = \partial/\partial x \tag{1.1}$$

has many remarkable properties, among them we are specially interested in the following.

1. Equations in the hierarchy have the bi-Hamiltonian structures and infinitely many conserved quantities (see [1, 2, 4]).

2. There exists a Miura transformation relating the equations to the modified equations. By the Miura transformation the second Hamiltonian structure of the n^{th} KdV type equations is transformed to a vastly simpler one (essentially just $\partial/\partial x$) on an appropriate space of the modified variables. This is what we call the Kupershmidt–Wilson (KW) theorem [5]. A short proof of this theorem was then given by Dickey [4, 6].

3. There exists a remarkable connection between the second Poisson brackets of the KdV type equations and the so-called W_n algebra in the conformal field theory. The Miura transformation plays an important role in the construction of the free field realization of the W algebra (see [7, 8] and references therein).