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A Functional Integral Approach to Shock Wave Solutions of Euler Equations with Spherical Symmetry

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Abstract: For $n \times n$ systems of conservation laws in one dimension without source terms, the existence of global weak solutions was proved by Glimm [1]. Glimm constructed approximate solutions using a difference scheme by solving a class of Riemann problems.

In this paper, we consider the Cauchy problem for the Euler equations in the spherically symmetric case when the initial data are small perturbations of the trivial solution, i.e., $u \equiv 0$ and $\rho \equiv \mathit{constant}$, where u is velocity and ρ is density. We show that this Cauchy problem can be reduced to an ideal nonlinear problem approximately. If we assume all the waves move at constant speeds in the ideal problem, by using Glimm's scheme and an integral approach to sum the contributions of the reflected waves that correspond to each path through the solution, we get uniform bounds on the L_∞ norm and total variational norm of the solutions for all time. The geometric effects of spherical symmetry leads to a non-integrable source term in the Euler equations. Correspondingly, we consider an infinite reflection problem and solve it by considering the cancellations between reflections of different orders in our ideal problem. Thus we view this as an analysis of the interaction effects at the quadratic level in a nonlinear model problem for the Euler equations. Although it is far more difficult to obtain estimates in the exact solutions of the Euler equations due to the problem of controlling the time at which the cancellations occur, we believe that this analysis of the wave behaviour will be the first step in solving the problem of existence of global weak solutions for the spherically symmetric Euler equations outside of fixed ball.

1. Introduction

We consider the Euler equations of the compressible gas dynamics in R^n,

$$\begin{aligned} &\rho_t + \nabla \cdot (\rho \vec{u}) = 0\,, \\ &(\rho \vec{u})_t + \nabla \cdot (\rho \vec{u} \otimes \vec{u} + P) = 0\,, \end{aligned} \tag{1}$$