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Poisson–Lie Group of Pseudodifferential Symbols

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Abstract: We introduce a Lie bialgebra structure on the central extension of the Lie algebra of differential operators (with scalar or matrix coefficients) on the line and on the circle. This defines a Poisson–Lie structure on the dual group of pseudodifferential symbols of an arbitrary real (or complex) order. We show that the usual (second) Benney, GL_n-KdV (or GL_n-Adler–Gelfand–Dickey) and KP Poisson structures are naturally realized as restrictions of this Poisson structure to submanifolds of this "universal" Poisson–Lie group. Moreover, the reduced ($=SL_n$) versions of these manifolds (or W_n-algebras in physical terminology) can be viewed as certain subspaces of the quotient of this Poisson–Lie group by the dressing action of the group of functions on the circle (or as a result of a Poisson reduction). Finally we define an infinite set of commuting functions on the Poisson–Lie group that give the standard families of Hamiltonians when restricted to the submanifolds mentioned above. The Poisson structure and Hamiltonians on the whole group interpolate between the Poisson structures and Hamiltonians of Benney, KP and KdV flows. We also discuss the geometrical meaning of W_∞ as a limit of Poisson algebras W_ε as $\varepsilon \to 0$.

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