

2 + 1-D Topological Quantum Field Theory and 2-D Conformal Field Theory

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Abstract: We describe the relation between three dimensional topological quantum field theory and two dimensional conformal field theory. Some applications to quantum knot invariants leading to the equivalence of Chern–Simons–Witten and Kohno's approaches are outlined.

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1. Introduction

After Witten [Wit89] introduced his invariants for 3-manifolds much work has been done on understanding them from the mathematical point of view. A counterpart to the Feynmann path integral formalism in the Chern–Simons theory has been given via quantum groups by Reshetikhin and Turaev [RT91]. The $SU(2)$ -theory has been extensively studied in [RT91, BHMV92, KM91, Koh92]. Recently the quantum group construction of invariants has been extended to the simple Lie groups in the series A, B, C, D by Turaev and Wenzl [TW93]. Several generalizations were given by Crane [Cra91] and Degiovanni [Deg92] which started from Rational Conformal Field Theories (abbrev. RCFT) in dimension 2 and derived Topological Quantum Field Theories (abbrev. TQFT) in dimension $2 + 1$. Also Kohno [Koh92] computed the mapping class group representation arising in the $SU(2)$ –WZW model and show how we can construct topological invariants from this data, by pointing out that these ideas work more generally for any RCFT.