Commun. Math. Phys. 171, 383 – 404 (1995)

Communications in Mathematical Physics

Differential Calculus on $ISO_q(N)$, Quantum Poincaré Algebra and q-Gravity

Leonardo Castellani

Istituto Nazionale di Fisica Nucleare, Sezione di Torino, and Dipartimento di Fisica Teorica, Via P. Giuria 1, I-10125 Torino, Italy

Received: 1 December 1993/in revised form: 11 November 1994

Abstract: We present a general method to deform the inhomogeneous algebras of the B_n, C_n, D_n type, and find the corresponding bicovariant differential calculus. The method is based on a projection from $B_{n+1}, C_{n+1}, D_{n+1}$. For example we obtain the (bicovariant) inhomogeneous q-algebra $ISO_q(N)$ as a consistent projection of the (bicovariant) q-algebra $SO_q(N+2)$. This projection works for particular multiparametric deformations of $SO(N+2)$, the so-called "minimal" deformations. The case of $ISO_q(4)$ is studied in detail: a real form corresponding to a Lorentz signature exists only for one of the minimal deformations, depending on one parameter q. The quantum Poincaré Lie algebra is given explicitly: it has 10 generators (no dilatations) and contains the *classical* Lorentz algebra. Only the commutation relations involving the momenta depend on q. Finally, we discuss a q-deformation of gravity based on the "gauging" of this q-Poincaré algebra: the lagrangian generalizes the usual Einstein–Cartan lagrangian.

1. Introduction

Perturbative quantum Einstein gravity is known to be mathematically inconsistent, since it is plagued by ultraviolet divergences appearing at two-loop order (the absence of one-loop divergencies was found in [1], whereas two-loop divergencies were explicitly computed in [2]). In supergravity the situation is only slightly better, the divergences starting presumably at three loops[1]. In the last fifteen years or so there have been various proposals to overcome this difficulty, and consistently quantize gravity either alone or as part of a unified theory of the fundamental interactions. Such a unified picture is provided by superstrings (see for a review [3]), where Einstein gravity arises as a low-energy effective theory, coupled more or less realistically to gauge fields and leptons, and regulated at the Planck scale by an

e-mail addresses: decnet= 31890::castellani;
internet= castellani@to.infn.it

[1] No explicit calculation like the one of ref. [2] exists, but there is no symmetry principle that excludes them.