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Inductive Limit Automorphisms of the Irrational Rotation Algebra

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Dedicated to Professor George Elliott on the occasion of his fiftieth birthday

Abstract: It is shown that the flip automorphism $U \mapsto U^*$, $V \mapsto V^*$ of the irrational rotation algebra A_θ is an inductive limit automorphism. Here, the algebra A_θ is generated by unitaries U, V satisfying $VU = e^{2\pi i\theta}UV$, where θ is an irrational number. Recently, Elliott and Evans proved that A_θ can be approximated by unital subalgebras isomorphic to a direct sum of two matrix algebras over $C(\mathbb{T})$, the algebra of continuous functions on the unit circle. This is the central result which they used to obtain their structure theorem on A_θ; namely, that A_θ is the inductive limit of an increasing sequence of subalgebras each isomorphic to a direct sum of two matrix algebras over $C(\mathbb{T})$. In their proof, they devised a subtle construction of two complementary towers of projections. In the present paper it is shown that the two towers can be chosen so that each summand of their approximating basic building blocks is invariant under the flip automorphism and, in particular, that the unit projection of the first summand is unitarily equivalent to the complement of the unit of the second by a unitary which is fixed under the flip. Also, an explicit computation of the flip on the approximating basic building blocks of A_θ is given. Further, combining this result along with others, including a theorem of Su and a spectral argument of Bratteli, Evans, and Kishimoto, a two-tower proof is obtained of the fact established by Bratteli and Kishimoto that the fixed point subalgebra B^θ (under the flip) is approximately finite dimensional. Also used here is the fact that B^θ has the cancellation property and is gifted with four basic unbounded trace functionals. The question is raised whether other finite order automorphisms of A_θ (arising from a matrix in $SL(2,\mathbb{Z})$) are inductive limit automorphisms – or even *almost* inductive limit automorphisms in the sense of Voiculescu.

1. Introduction

In a recent paper, Elliott and Evans [6] devised a subtle construction of two towers of projections in their remarkable proof of the structure theorem on the irrational rotation algebra A_θ. This result states that A_θ is the inductive limit of a sequence of direct sums of two matrix algebras over $C(\mathbb{T})$, the algebra of continuous functions

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