

# Magnetic Lieb–Thirring Inequalities

**László Erdős\***

Department of Mathematics, Princeton University, Princeton, NJ 08544, USA.  
E-mail: erdlac@math.princeton.edu. Fax: (609)-258-1367

Received: 9 May 1994

**Abstract:** We study the generalizations of the well-known Lieb–Thirring inequality for the magnetic Schrödinger operator with nonconstant magnetic field. Our main result is the naturally expected magnetic Lieb–Thirring estimate on the moments of the negative eigenvalues for a certain class of magnetic fields (including even some unbounded ones). We develop a localization technique in path space of the stochastic Feynman–Kac representation of the heat kernel which effectively estimates the oscillatory effect due to the magnetic phase factor.

## Contents

1. Introduction . . . . .	630
2. Definitions, Conjectures, Results . . . . .	631
2.1. Magnetic Field with Constant Direction . . . . .	631
2.2. Magnetic Field with Arbitrary Direction . . . . .	635
3. Separation of the External Potential . . . . .	637
4. Feynman–Kac Formulas . . . . .	642
4.1. Two-Dimensional Case . . . . .	642
4.2. Three-Dimensional Case . . . . .	644
5. Bounded Magnetic Field with Weak Singularities . . . . .	647
6. Unbounded Magnetic Field; Reduction to the Main Lemma . . . . .	648
7. Proof of the Main Lemma . . . . .	653
7.1. A Sequence of Stopping Times . . . . .	654
7.2. Estimating $I_0$ (No Reflection) . . . . .	655
7.3. Estimating $I_n$ for $n \geq 1$ Using Reflections . . . . .	659
A. Selfadjointness and Negative Essential Spectrum . . . . .	664
B. Counterexample . . . . .	665
References . . . . .	667

---

\* Work supported by the NSF grant PHY90-19433 A02, by the Alfred Sloan Foundation dissertation Fellowship and by the Erwin Schrödinger Institute for Mathematical Physics in Vienna.