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Spectral Families of Quantum Stochastic Integrals

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Abstract: We develop a theory of spectral integration for quantum stochastic integrals of certain families of processes driven by creation, conservation and annihilation processes in Fock space. These give a non-commutative generalisation of classical stochastic integrals driven by Poisson random measures. A stochastic calculus for these processes is developed and used to obtain unitary operator valued solutions of stochastic differential equations. As an application we construct stochastic flows on operator algebras driven by Lévy processes with finite Lévy measure.

1. Introduction

The standard theory of quantum stochastic calculus in boson Fock space as developed by R.L. Hudson and K.R. Parthasarathy uses the creation, conservation and annihilation processes as basic martingales (see [HuPa 1], [Par], [Mey] and references therein) and it is well known that classical stochastic integrals with respect to a standard Brownian motion or compensated Poisson process can be constructed within this more general formalism. As it stands however the theory is insufficiently fine to capture stochastic integrals with respect to Poisson random measures. The aim of this paper is to take the first steps towards developing the required generalisation.

Of course stochastic integrals with respect to Poisson random measures involve two distinct integrations over both time and space variables. In the quantum case we find that integration over the "space" variable corresponds to a spectral integral and hence the objects which we need to make sense of are operators of the form

$$M(t) = \int_{-\infty}^{\infty} \int_{0}^{t} \{E_1(s,\lambda) A_x^{\dagger}(ds, P(d\lambda)) + E_2(s,\lambda) \Lambda(ds, P(d\lambda)) + E_3(s,\lambda) A_y(ds, P(d\lambda))\} , \qquad (1.1)$$

where P is a projection-valued measure, $E_j\,(j = 1,2,3)$ are suitable families of adapted processes and $A^{\dagger}, \Lambda$ and A represent the differentials of the creation, conservation and annihilation processes (respectively) in a sense which is made precise