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On the support of the Ashtekar-Lewandowski Measure

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Abstract: We show that the Ashtekar-Isham extension $\overline{\mathscr{A}/\mathscr{G}}$ of the configuration space of Yang-Mills theories $\mathscr{A}/\mathscr{G}$ is (topologically and measure-theoretically) the projective limit of a family of finite dimensional spaces associated with arbitrary finite lattices.

These results are then used to prove that $\mathscr{A}/\mathscr{G}$ is contained in a zero measure subset of $\overline{\mathscr{A}/\mathscr{G}}$ with respect to the diffeomorphism invariant Ashtekar-Lewandowski measure on $\overline{\mathscr{A}/\mathscr{G}}$. Much as in scalar field theory, this implies that states in the quantum theory associated with this measure can be realized as functions on the "extended" configuration space $\overline{\mathscr{A}/\mathscr{G}}$.

1. Introduction

The usual canonical approach to quantization of a (finite dimensional) system defines states as functions on a configuration space and defines an inner product of two such functions ψ and ϕ through

$$(\psi, \phi) = \int_{\mathscr{Q}} d\mu \; \psi^* \phi,$$

where μ is some measure on the configuration space $\mathscr{Q}$. Naively applying this procedure to Yang-Mills theories produces a "connection representation" with states that are functions of the Yang-Mills connection. In particular, these states are functions on the quotient space $\mathscr{A}/\mathscr{G}$, where $\mathscr{A}$ is the space of (C^1-)connections and $\mathscr{G}$ is the group of (C^2-)gauge transformations. The same is true for gravity formulated in terms of Ashtekar variables before one imposes the diffeomorphism and hamiltonian constraints [1,2].

A more sophisticated analysis of examples, such as scalar field theory [3-5], shows that the domain space of the wave functions may not be exactly the classical configuration space. Instead, some extension of $\mathscr{Q}$ is required.

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