Prescribing Topological Defects for the Coupled Einstein and Abelian Higgs Equations

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Abstract: We construct multi-string solutions of the coupled Einstein and Abelian Higgs equations so that the spacetime is uniform along the time axis and a vertical direction and nontrivial geometry is coded on a Riemann surface M. We concentrate on the critical Bogomol'nyi phase. When M is compact, the Abelian Higgs model is defined by a complex line bundle L over M. We prove that, due to the coupling of the Einstein equations, the Euler characteristic of M and the first Chern number of the line bundle L identified as the total string number impose an exact obstruction to the existence of a string solution. Such an obstruction leads to some interesting implications. We then study the existence of multi-string solutions which can realize a prescribed string distribution. We show that there are such solutions when the local string winding numbers do not exceed half of the total string number. When M is noncompact and globally conformal to a plane, we show that the energy scale of symmetry breaking plays a crucial role and there are finite-energy radially symmetric string solutions realizing a given string number if and only if the symmetry breaking scale is sufficiently small but nonvanishing. Finally, we obtain finite-energy multi-string solutions with an arbitrary string distribution and associated local winding numbers. These solutions are not radially symmetric and are regular everywhere and topologically nontrivial so that both the energy of the matter-gauge sector and the energy of the gravitational sector viewed as the total Gauss curvature of M are quantized.

0. Introduction

Domain walls, strings, and monopoles are interesting topological defects arising as static solutions of gauge field equations with broken symmetry and nontrivial topology. When the Einstein equations are coupled into the theory, these solutions give rise to various cosmological implications. Cosmic strings are static solutions of the coupled Einstein and Yang–Mills–Higgs equations so that the spacetime is uniform

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