Commun. Math. Phys. 170, 529 – 539 (1995)

Communications in Mathematical Physics

Renormalization Group Fixed Points in the Local Potential Approximation for $d \geqq 3$

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Received: 1 March 1994/in revised form: 30 September 1994

Abstract: In the Local Potential Approximation, renormalization group equations reduce to a semilinear parabolic partial differential equation. Felder [8] has derived this equation and has constructed a family of non-trivial fixed points u^*_{2n} $(n = 2, 3, 4, \ldots)$ which have the form of n-well potentials and exist in the ranges of dimensions $2 < d < 2 + \frac{2}{n-1}$. In this paper we show that if $d \geqq 4$, then these non-trivial fixed points disappear, and if $3 \leqq d < 4$ then we have only the u^*_4 fixed point.

1. Introduction

Non-trivial fixed points of the renormalization group (RG) play a crucial role in the understanding of statistical mechanical systems in the vicinity of the critical point [1, 2]. In the case of a symmetric scalar field the non-trivial fixed points are expected to appear as bifurcating from the trivial massless fixed point as one varies continuously the dimension d of the space [3]. These bifurcations occur at thresholds $d_n = 2 + \frac{2}{n-1}$, $n = 2, 3, \ldots$, where the linearized RG acquires a zero mode (the fixed point which appears at $d = d_n$ is called the ϕ^{2n}_d fixed point and it looks like a $n - well$ potential). This pattern is not well understood from the mathematical point of view, but some pieces of it were established in toy models like Dyson's one [4]: Bleher and Sinai [5] proved the existence of a non-trivial fixed point if $d = d_n - \varepsilon$, where $\varepsilon > 0$ is small enough. Felder [8] showed that in the Local Potential Approximation the ϕ^{2n}_d fixed point exists for $2 < d < d_n$.

It is believed that the ϕ^{2n}_d fixed points disappear for $d \geqq d_n$. Gawedzki and Kupiainen [6] showed that if $d \geqq 4$ and the potential is even and "small" (weak coupling) then, under the flow of the RG transformation, it is driven to the Gaussian fixed point. Aizenman [7] showed in $d > 4$ dimensions that the (even) ϕ^d_4 Euclidean field theory, with a cut-off, is inevitably free in the continuum limit. In this paper we show that in the Local Potential Approximation if $d \geqq 4$ there is no non-trivial fixed point and if $3 \leqq d < 4$ the only non-trivial fixed point is the one which appears at $d = 4$, the ϕ^4_d fixed point.

*Research supported by CNPq, Brazil