Commun. Math. Phys. 170, 483 – 502 (1995)

Communications in Mathematical Physics

A Phase Space for Gravitational Radiation

Adam D. Helfer
Department of Mathematics, University of Missouri, Columbia, MO 65211, U.S.A.

Received: 6 July 1993/in revised form: 28 March 1994

Abstract: We give a new definition, based on considerations of well-posedness for a certain asymptotic initial value problem, of the phase space for the radiative degrees of freedom of the gravitational field in exact General Relativity. This space fibres over the space of final states, with the fibres being the purely radiative degrees of freedom. The symplectic form is rigorously identified.

The infrared sectors are shown to be the level surfaces of a moment map of an action of the quotient group Supertranslations/Translations. A similar result holds for Electromagnetism in Minkowski space.

1. Introduction

The theory of gravitational radiation interweaves physical and mathematical progress to an unusual degree: some physical intuition is necessary to begin to pose questions which are significant; and the construction of a mathematically satisfactory theory often serves as a test of that intuition. If we define radiation mathematically as disturbances which escape to (or come in from) infinity, then gravitational radiation, which travels at the speed of light, will escape to null infinity, and ought to be analyzed there. Since what "null" is determined by the dynamical field, from the analytic point of view one is studying the asymptotics along characteristics of a system of quasilinear equations. There are several, inequivalent, mathematical meanings this might have, and the nature of the physical questions under consideration must guide the analysis.

It was Bondi and coworkers (Bondi 1960, Bondi, van der Burg and Metzner 1962, Sachs 1962a; see also Newman and Penrose 1962) who introduced the idea of analyzing the field at large null separations, and Penrose (1963) who recast their asymptotic conditions as the existence of a null hypersurface $\mathscr{I}$ at null infinity. After this work, it was clear that at a formal level the radiative degrees of freedom were represented by a function, now called the Bondi shear, on $\mathscr{I}$.

Ashtekar and Streubel (1981) seem to have been the first to realize that it would be desirable to develop, not just a space-time by space-time analysis, but a phase space of radiative degrees of freedom in terms of the asymptotics of the fields.