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Complete Analyticity for 2D Ising Completed

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Abstract: We study the behavior of the two-dimensional nearest neighbor ferromagnetic Ising model under an external magnetic field h. We extend to every subcritical value of the temperature a result previously proven by Martirosyan at low enough temperature, and which roughly states that for finite systems with – boundary conditions under a positive external field, the boundary effect dominates in the bulk if the linear size of the system is of order B/h with B small enough, while if B is large enough, then the external field dominates in the bulk. As a consequence we are able to complete the proof that "complete analyticity for nice sets" holds for every value of the temperature and external field in the interior of the uniqueness region in the phase diagram of the model.

The main tools used are the results and techniques developed to study large deviations for the block magnetization in the absence of the magnetic field, and recently extended to all temperatures below the critical one by Ioffe.

1. Introduction

In this paper we consider the two-dimensional ferromagnetic Ising model with homogeneous interaction between nearest neighbors and subject to a homogeneous external field, i.e., the statistical mechanics model on $\mathbb{Z}^2$ with formal Hamiltonian

$$H_h(\sigma) = -\frac{1}{2} \sum_{x,y \text{ n.n.}} \sigma(x)\sigma(y) - \frac{h}{2} \sum_x \sigma(x) , \tag{1.1}$$

where $\sigma(x) = \pm 1$ is the spin at the site $x \in \mathbb{Z}^2$, and the first sum runs over pairs of sites which are nearest neighbors in $\mathbb{Z}^2$, each pair counted only once. Precise definitions, along with notation will be provided in the next section, but below we describe in somewhat informal terms, what will be done.

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