Commun. Math. Phys. 170, 445 – 451 (1995)

Communications in Mathematical Physics



# A Note on the Index Bundle over the Moduli Space of Monopoles

**John D. S. Jones**[1], **Michael K. Murray**[2]

[1] Mathematics Institute, University of Warwick, Coventry CV 4 7AL, United Kingdom. jdsj @maths.warwick.ac.uk

[2] Department of Pure Mathematics, The University of Adelaide, Adelaide, SA 5005 Australia. mmurray @maths.adelaide.edu.au

Received: 28 July 1994/in revised form: 25 October 1994

**Abstract:** Donaldson has shown that the moduli space of monopoles $M_k$ is diffeomorphic to the space $Rat_k$ of based rational maps from the two-sphere to itself. We use this diffeomorphism to give an explicit description of the bundle on $Rat_k$ obtained by pushing out the index bundle from $M_k$. This gives an alternative and more explicit proof of some earlier results of Cohen and Jones.

## 1. Introduction

In [4] Cohen and Jones study the topological type of the index bundle of various families of Dirac operators arising in the theory of monopoles and the relation between these index bundles and representations of the braid groups. The methods used in this general study were those of algebraic topology and index theory. For example, it was shown that using Donaldson's diffeomorphism between monopoles and based rational maps [5] and the relation between the space of based rational maps and the braid group that the $K$-theory class of the index bundle over the space of monopoles is completely determined by representations of the braid groups. In this note we show how Donaldson's diffeomorphism gives rise to a simple explicit characterisation of the bundle over the space of rational maps corresponding to this index bundle. The corresponding representation of the braid group is readily identified.

In more detail, let $M_k$ denote the moduli space of framed monopoles of charge $k$ over $\mathbf{R}^3$ with structure group $SU(2)$. Donaldson in [5] defines an explicit diffeomorphism

$$M_k \to Rat_k \tag{1.1}$$

from $M_k$ to the space of all based rational maps of degree $k$ from the two-sphere to itself. It can be shown [10] that if $(A, \Phi)$ is a monopole, then the space of $L^2$ solutions of the Dirac equation coupled to $(A, \Phi)$ has dimension $k$, where $k$ is the charge of the monopole. This defines a complex vector bundle over $M_k$ which, in fact, has a hermitian inner product and a real structure and hence has structure group $O(k)$, the group of real, orthogonal, $k$ by $k$ matrices. We denote by $Ind_k$, the corresponding principal $O(k)$ bundle on $M_k$. This is the index bundle and