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Entropic Repulsion of the Lattice Free Field

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Abstract: Consider the massless free field on the d-dimensional lattice $\mathbb{Z}^d, d \geqq 3$; that is the centered Gaussian field on $\mathbb{R}^{\mathbb{Z}^d}$ with covariances given by the Green function of the simple random walk on $\mathbb{Z}^d$. We show that the probability, that all the spins are positive in a box of volume N^d, decays exponentially at a rate of order $N^{d-2} \log N$ and compute explicitly the corresponding constant in terms of the capacity of the unit cube. The result is extended to a class of transient random walks with transition functions in the domain of the normal and α-stable law.

1. Introduction and Result

Let $Q = \{Q(k,j), k,j \in \mathbb{Z}^d\}$ be the transition matrix of a symmetric *transient* random walk on the d-dimensional lattice $\mathbb{Z}^d$. More specifically we will be interested in two types of situations:

(a) $d \geqq 3$, Q is the transition function of the simple random walk:

$$Q(i,k) = \begin{cases} \frac{1}{2d} & \text{if } |i-k| = 1\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

(b) $d \geqq 1, q_\alpha$ is the density of the symmetric isotropic α-stable law on $\mathbb{R}^d$ for some $0 < \alpha < 2 \wedge d$, see (A.1),

$$Q(i,k) = \int_V q_\alpha(x + (i-k)^+)\, dx\,,$$

where $V = [-\frac{1}{2}, \frac{1}{2}]^d$, $(j)^+ = (|j_1|, |j_2|, \ldots, |j_d|)$.

Let $G = \sum_{n=0}^{\infty} Q^n$ be the corresponding Green function. Then it is well known that

$$\lim_{|k-j| \to \infty} \frac{G(j,k)}{g_\alpha(j-k)} = 1\,,$$

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