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# On Algebraic Equations Satisfied by Hypergeometric Correlators in WZW Models. II.

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**Abstract:** The paper contains an explicit description of genus 0 conformal block bundles for Wess–Zumino–Witten models of Conformal field theory. We prove that an earlier construction due to the second and the third authors gives a map of these bundles to certain de Rham cohomology bundles.

## 1. Introduction

*1.1.* Let $\mathfrak{g}$ be a simple finite dimensional complex Lie algebra; let $( \ , \ )$ be an invariant scalar product on $\mathfrak{g}$ normalized in such a way that $(\theta, \theta) = 2$, $\theta$ being the highest root. Fix a positive integer $k$. Let $L_1, \dots, L_{n+1}$ be irreducible representations of $\mathfrak{g}$ with highest weights $\Lambda_1, \dots, \Lambda_{n+1}$. Suppose that $(\Lambda_i, \theta) \leqq k$ for all $i$.

Consider a complex affine $n$-dimensional affine space $\mathbb{A}^n$ with fixed coordinates $\mathbf{z} = (z_1, \dots, z_n)$. Consider the space $X_n = \mathbb{A}^n - \bigcup_{i,j} \Delta_{ij}$, where $\Delta_{ij} = \{(z_1, \dots, z_n) | z_i = z_j\}$ are diagonals. According to Conformal field theory, one can define a remarkable finite dimensional holomorphic vector bundle $\mathscr{C}(\Lambda_1, \dots, \Lambda_{n+1})$ over $X_n$ equipped with a flat connection (with logarithmic singularities along $\Delta_{ij}$). (We imply that the last representation "lives" at the point $z_{n+1} = \infty$.)

More precisely, consider a trivial bundle over $X_n$ with a fiber $(L_1 \otimes \dots \otimes L_{n+1})_{\mathfrak{g}}$. Here we denote by $M_{\mathfrak{g}}$ the space of coinvariants $M/\mathfrak{g}M$ of a $\mathfrak{g}$-module $M$. Let us denote this bundle by $\mathscr{B}(\Lambda_1, \dots, \Lambda_{n+1})$; it is equipped with a flat connection given by a system of Knizhnik–Zamolodchikov (KZ) differential equations, [KZ]. The bundle $\mathscr{C}(\Lambda_1, \dots, \Lambda_{n+1})$ is a certain quotient of $\mathscr{B}(\Lambda_1, \dots, \Lambda_{n+1})$ stable under KZ connection.

Classically this quotient is described in terms of certain coinvariants of the tensor product $\mathbf{L}_1 \otimes \dots \otimes \mathbf{L}_{n+1}$, where $\mathbf{L}_i$ is the irreducible representation of the affine Kac–Moody algebra $\hat{\mathfrak{g}}$ corresponding to $L_i$ and having the central charge $k$ (see for example [KL] or Sect. 2 below). The first goal of the present paper is a precise

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