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Quantum Principal Commutative Subalgebra in the Nilpotent Part of $U_q\widehat{sl}_2$ and Lattice KdV Variables

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Abstract: We propose a quantum lattice version of B. Feigin and E. Frenkel's constructions, identifying the KdV differential polynomials with functions on a homogeneous space under the nilpotent part of $\widehat{sl}_2$. We construct an action of the nilpotent part $U_q\widehat{n}_+$ of $U_q\widehat{sl}_2$ on their lattice counterparts, and embed the lattice variables in a $U_q\widehat{n}_+$-module, coinduced from a quantum version of the principal commutative subalgebra, which is defined using the identification of $U_q\widehat{n}_+$ with its dual algebra.

Introduction

In [FF1, FF2], B. Feigin and E. Frenkel propose a new approach to the generalized KdV hierarchies. They construct an action of the nilpotent part $\widehat{n}_+$ of the affine algebra $\widehat{g}$ on differential polynomials in the Miura fields, connected to the action of screening operators. This enables them to consider these differential polynomials as functions on a homogeneous space of $\widehat{n}_+$, and to interpret in this way the KdV flows. They also suggest that analogous constructions should hold for the quantum KdV equations.

In this work we propose a quantum lattice version of part of these constructions. Following ideas of lattice W-algebras, we replace the differential polynomials by an algebra of q-commuting variables, set on a half-infinite line. The analogue of the action of [FF1] is then an action of the nilpotent part $U_q\widehat{n}_+$ of the quantum affine algebra $U_q\widehat{sl}_2$. Recall that the homogeneous space occurring in [FF1] is $\widehat{N}_+/A$, where $\widehat{N}_+$ and A are the groups corresponding to $\widehat{n}_+$ and its principal commutative subalgebra a. A natural question is then what the analogue of a is in the quantum situation.

We construct a quantum analogue of a in the following way: we use an isomorphism of $U_q\widehat{b}_+$ with the coordinate ring $\mathbf{C}[\widehat{B}_+]_q$ ([Dr, LSS]) and transport in the first algebra a twisted version of the well-known commutative family res $d\lambda\lambda^k \operatorname{tr} T(\lambda)$. We prove that this subalgebra of $U_q\widehat{b}_+$ gives Ua for $q = 1$. This proof uses characterizations of these algebras as centralizers of one element.