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Quasifinite Highest Weight Modules over the Super $\mathcal{W}_{1+\infty}$ Algebra

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Abstract: We study quasifinite highest weight modules over the supersymmetric extension of the $\mathcal{W}_{1+\infty}$ algebra on the basis of the analysis by Kac and Radul. We find that the quasifiniteness of the modules is again characterized by polynomials, and obtain the differential equations for highest weights. The spectral flow, free field realization over the (B,C)-system, and the embedding into $\widehat{\mathrm{gl}}(\infty|\infty)$ are also presented.

1. Introduction

Conformal field theory has attracted much interest for the last ten years, since it describes classical vacua of string theory and the two-dimensional statistical system at fixed points of the renormalization group. The representation theory of the Virasoro algebra plays a central role [BPZ]. However, when the systems have larger symmetries, the Virasoro algebra must be extended. For example, when supersymmetry exists one will be led to the super Virasoro algebra [NS, R, GS], while for $\mathbb{Z}_N$ symmetry what is called the $\mathcal{W}_N$ algebra will be relevant [Z, BS].

In the $\mathcal{W}_N$ algebra (or its supersymmetric extension [IMY]), there are $(N-1)$ generating currents with spins $s = 2,3, \ldots, N$ (and their superpartners if supersymmetry exists). Here $s = 2$ corresponds to the energy momentum tensor. The peculiar nature of the algebra is in its nonlinearity, *i.e.,* the singular part of the operator product of generating currents is not expanded as a linear combination of the generating currents, and one has to introduce composite fields made of the currents. The occurrence of such operators implies that the corresponding algebra

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