Commun. Math. Phys. 170, 1 – 20 (1995)

Communications in Mathematical Physics

Symmetries of Quantum Spaces. Subgroups and Quotient Spaces of Quantum $SU(2)$ and $SO(3)$ Groups

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Received: 9 November 1992 / in revised form: 18 February 1994

Abstract: We prove that each action of a compact matrix quantum group on a compact quantum space can be decomposed into irreducible representations of the group. We give the formula for the corresponding multiplicities in the case of the quotient quantum spaces. We describe the subgroups and the quotient spaces of quantum $SU(2)$ and $SO(3)$ groups.

0. Introduction

Quantum groups have been already applied in various areas of physics, like conformal field theory and exactly solvable models in statistical mechanics. It is especially interesting that they could possibly describe symmetries of (quantum) space-time in a future quantum gravity. In the same time, the nature and properties of quantum groups are still under investigation. The local description of quantum groups is given in terms of quantum universal enveloping algebras (cf. e.g. [Dr, J]). In the global description we investigate the functions on quantum groups (cf. e.g. [W2, RTF]). A deep insight in that global structure is given by the topological approach developed in the series of papers of S.L. Woronowicz [W1-W6]. We use that approach in the present paper.

The classical $SU(2)$ and $SO(3)$ groups play an important role in description of spherically symmetric, stationary problems in physics. Also their subgroups are important in description of various physical systems. The description of quantum $SU(2)$ groups was given in [W2]. Their quantum homogeneous spaces, quantum 2-spheres, were investigated in [P1, P2, P5] (cf. also [VS2]). However, the general theory of quantum subgroups and quantum homogeneous spaces was only touched there. In the present paper we want to treat that subject in more detail. We also provide more examples.

In Sect. 1 we investigate the general theory of the (right) actions of (compact matrix) quantum groups on (compact) quantum spaces. In Sect. 2 and 3 the theory is illustrated on the example of quantum $SU(2)$ and $SO(3)$ groups. We classify their subgroups and describe the corresponding quotient spaces. Provided examples of finite quantum groups can have an application in the theory of pseudogroups of