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On Separation of Phases in One-Dimensional Gases

Kurt Johansson

Department of Mathematics, Royal Institute of Technology, S-10044 Stockholm, Sweden.
e-mail: kurtj@math.kth.se

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Abstract: We prove that in a one-dimensional gas in the canonical ensemble with pair interaction $A/r^{\gamma} - B/r^2$, $\gamma > 2$, we have a separation of phases at sufficiently low temperatures. The same combinatorial framework can be used for both lattice and continuous models. A rather precise bound on the critical temperature in a $1/r^2$ lattice gas is obtained when the nearest neighbour coupling is large. The interface between the two phases is defined and investigated.

0. Introduction

There are many results on phase transitions in lattice models in all dimensions. For continuous models however there are very few results, see [Ru2] or [Is], appendix. It is still an open problem to prove that there is a phase transition in a 3-dimensional continuous gas with a Lennard-Jones interaction, $A/r^{12} - B/r^6$, see [Si], problem 7. In view of this it seems worthwhile to try to establish the existence of a phase transition in a 1-dimensional continuous gas in particular in the non-hard core case. A phase transition in one dimension requires a long-range interaction, which makes the argument complicated, but on the other hand the difference between a lattice and a continuous gas should be least in one dimension.

We need an energy-entropy argument which is not so sensitive to the exact location of the particles, i.e. if we move the particles slightly the argument should still be valid. In two previous papers such a method was developed and it was proved that in a lattice gas [Jo1] or in a hard-core continuous gas [Jo2], in the canonical ensemble, with attractive pair interaction $-1/r^{\alpha}$, $1 < \alpha \leq 2$, there is condensation at sufficiently low temperatures. Here condensation means that for a large system, with high probability, the gas has non-uniform density, a separation of phases occurs. A heuristic argument is given in the introduction to [Jo1].

In this paper these results will be extended to a continuous gas without hardcore. The pair interaction will be of Lennard-Jones type, $A/r^{\gamma} - B/r^2$, $\gamma > 2$. This model is more unlike a lattice gas and is more difficult to deal with than the hard-core continuous gas.