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Elliptic Quantum Many-Body Problem and Double Affine Knizhnik–Zamolodohikov Equation

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Abstract: The elliptic-matrix quantum Olshanetsky–Perelomov problem is introduced for arbitrary root systems by means of an elliptic version of the Dunkl operators. Its equivalence with the double affine generalization of the Knizhnik–Zamolodchikov equation (in the induced representations) is established.

0. Introduction

We generalize the affine Knizhnik–Zamolodchikov equation from [Ch1,2,3] replacing the corresponding root systems by their affine counterparts. To explain the construction in the case of the root system of $\mathbf{gl}_n$, let us first introduce the **affine Weyl group** $\mathbf{S}_n^a$. It is the semi-direct product of the symmetric group $\mathbf{S}_n$ and the lattice $A = \oplus_{i=1}^{n-1} \mathbf{Z}\varepsilon_{ii+1}$, where the first acts on the second permuting $\{\varepsilon_i, \varepsilon_{ij} = \varepsilon_i - \varepsilon_j\}$ naturally. This group is generated by the adjacent transpositions

$$s_i = (ii+1),\ 1 \leqq i < n,\ \text{and}\ s_0 = s_{n1}^{[1]},\ \text{where}\ s_{ij}^{[k]} = (ij)(k\varepsilon_{ij}) \in \mathbf{S}_n^a\,.$$

Setting

$$s_{ij}^{[k]}(b) = b - (\varepsilon_{ij}, b)(\varepsilon_{ij} + kc),\ s_{ij}^{[k]}(c) = c,\ \ b \in B = \bigoplus_{i=1}^n \mathbf{Z}\varepsilon_i\,, \qquad (0.1)$$

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