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Scaling for a Random Polymer

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Abstract: Let Q_n^β be the law of the n-step random walk on $\mathbb{Z}^d$ obtained by weighting simple random walk with a factor $e^{-\beta}$ for every self-intersection (Domb-Joyce model of "soft polymers"). It was proved by Greven and den Hollander (1993) that in $d = 1$ and for every $\beta \in (0, \infty)$ there exist $\theta^*(\beta) \in (0, 1)$ and $\mu_\beta^* \in \{\mu \in l^1(\mathbb{N}) : \|\mu\|_{l^1} = 1, \mu > 0\}$ such that under the law Q_n^β as $n \to \infty$:

(i) $\theta^*(\beta)$ is the limit empirical speed of the random walk;
(ii) μ_β^* is the limit empirical distribution of the local times.

A representation was given for $\theta^*(\beta)$ and μ_β^* in terms of a largest eigenvalue problem for a certain family of $\mathbb{N} \times \mathbb{N}$ matrices. In the present paper we use this representation to prove the following scaling result as $\beta \downarrow 0$:

(i) $\beta^{-\frac{1}{3}}\theta^*(\beta) \to b^*$;
(ii) $\beta^{-\frac{1}{3}}\mu_\beta^*(\lceil \cdot \, \beta^{-\frac{1}{3}} \rceil) \to^{L^1} \eta^*(\,\cdot\,)$.

The limits $b^* \in (0, \infty)$ and $\eta^* \in \{\eta \in L^1(\mathbb{R}^+) : \|\eta\|_{L^1} = 1, \eta > 0\}$ are identified in terms of a Sturm-Liouville problem, which turns out to have several interesting properties.

The techniques that are used in the proof are functional analytic and revolve around the notion of epi-convergence of functionals on $L^2(\mathbb{R}^+)$. Our scaling result shows that the speed of soft polymers in $d = 1$ is not right-differentiable at $\beta = 0$, which precludes expansion techniques that have been used successfully in $d \geqq 5$ (Hara and Slade (1992a, b)). In simulations the scaling limit is seen for $\beta \leqq 10^{-2}$.

0. Introduction and Main Results

0.1. Model and Motivation. A polymer is a long chain of molecules with two characteristic properties: (i) an irregular shape (due to entanglement); (ii) a certain stiffness (due to sterical hindrance). One way of describing such a polymer is the following model, which is based on a *random walk with self-repellence.*