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The Additivity of the η-Invariant. The Case of a Singular Tangential Operator

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Abstract: We prove the decomposition formula for the η-invariant of the compatible Dirac operator on a closed manifold M which is a sum of two submanifolds with common boundary.

0. Introduction

Let M be a compact odd-dimensional Riemannian manifold without boundary. Let $A : C^\infty(S) \to C^\infty(S)$ denote a compatible Dirac operator acting on sections of a bundle of Clifford modules S over M (see [6,8]). Then A is a self-adjoint elliptic operator. It has a discrete spectrum $\{\lambda_k\}_{k\in\mathbf{Z}}$. We define the eta function of the operator A as follows:

$$\eta(A;s) = \sum_{\lambda_k \neq 0} \text{sign } (\lambda_k)|\lambda_k|^{-s} . \tag{0.1}$$

Now $\eta(A;s)$ is a holomorphic function of s for $Re(s) > dim(M)$, and it has a meromorphic extension to $\mathbf{C}$, with isolated simple poles on the real axis and locally computable residue (see [1,8,13]). In particular, we know that if A is a compatible Dirac operator, then $\eta(A;s)$ is holomorphic for $Re(s) > -2$. The value of $\eta(A;s)$ at $s = 0$ is an important invariant of the operator, the bundle, and the manifold. We call $\eta(A;0)$ the eta invariant of A and denote it by η_A. We use the heat representation for the eta function and obtain the following formula for η_A:

$$\eta_A = \frac{1}{\sqrt{\pi}} \cdot \int_0^\infty \frac{1}{\sqrt{t}} \cdot Tr(Ae^{-tA^2})dt . \tag{0.2}$$

In this paper we study the decomposition of η_A into the contributions coming from different parts of the manifold M. The problem here is that η_A is not given by the local formula and it depends on the global geometry of the manifold and the operator (see [1,13]). Therefore it is somewhat surprising that we can present a satisfactory result.

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