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The Semiclassical Limit for $SU(2)$ and $SO(3)$ Gauge Theory on the Torus

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Abstract: We prove that for $SU(2)$ and $SO(3)$ quantum gauge theory on a torus, holonomy expectation values with respect to the Yang-Mills measure $d\mu_T(\omega) = N_T^{-1}e^{-S_{YM}(\omega)/T}[\mathscr{D}\omega]$ converge, as $T \downarrow 0$, to integrals with respect to a symplectic volume measure μ_0 on the moduli space of flat connections on the bundle. These moduli spaces and the symplectic structures are described explicitly.

1. Introduction

In this paper we prove that for $SU(2)$ and $SO(3)$ quantum gauge theory on a torus, the holonomy ("Wilson loop") expectation values for the Yang-Mills measure $d\mu_T(\omega) = N_T^{-1}e^{-S_{YM}(\omega)/T}[\mathscr{D}\omega]$ (the notation is explained in Subsect. 2.5 below) converge, as $T \downarrow 0$, to integrals with respect to a symplectic volume measure μ_0 on the moduli space of flat connections on the bundle. We also show that for the non-trivial $SO(3)$–bundle over the torus, the moduli space of flat connections consists of just one point and the limiting measure exists and is thus, of course, just the unit mass on this point (a similar situation exists in genus 0, which is treated from a slightly different point of view in [Se 1]). The proofs are by direct computation using the expectation value formulas derived from a continuum quantum gauge theory in [Se 2, 3] and by lattice theory in a number of works including [Wi 1] (the work [Wi 2] also contains results of related interest), and the description of the symplectic form obtained in [KS 1].

The most significant result related to the present work is the corresponding result by Forman [Fo] for gauge theory on compact orientable surfaces of genus > 1. Forman's proof relies on results of Witten [Wi 1]; a more direct proof of part of Forman's result has been obtained by C. King and the author in [KS 2]. The main case we work with in this paper, genus 1 and gauge group $SU(2)$, is singular in two ways (thereby making the method used in [Fo, Wi, KS 2] inapplicable): (1) the "partition function" goes to ∞, as $T \downarrow 0$, and (2) no flat connection is irreducible. The situation over the torus is singular for other gauge groups as well, but the case

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