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Strong Magnetic Fields, Dirichlet Boundaries, and Spectral Gaps

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Abstract: We consider magnetic Schrödinger operators

$$H(\lambda\vec{a}) = (-i\nabla - \lambda\vec{a}(x))^2$$

in $L_2(\mathbf{R}^n)$, where $\vec{a} \in C^1(\mathbf{R}^n; \mathbf{R}^n)$ and $\lambda \in \mathbf{R}$. Letting $M = \{x; B(x) = 0\}$, where B is the magnetic field associated with $\vec{a}$, and $M_{\vec{a}} = \{x; \vec{a}(x) = 0\}$, we prove that $H(\lambda\vec{a})$ converges to the (Dirichlet) Laplacian on the closed set M in the strong resolvent sense, as $\lambda \to \infty$, *provided* the set $M \backslash M_{\vec{a}}$ has measure zero.

In various situations, which include the case of periodic fields, we even obtain norm resolvent convergence (again under the condition that $M \backslash M_{\vec{a}}$ has measure zero). As a consequence, if we are given a periodic field B where the regions with $B = 0$ have non-empty interior and are enclosed by the region with $B \neq 0$, magnetic wells will be created when λ is large, opening up gaps in the spectrum of $H(\lambda\vec{a})$.

We finally address the question of absolute continuity of $H(\vec{a})$ for periodic $\vec{a}$.

0. Introduction

While the resolvent-limit of the Schrödinger operators $-\Delta + \lambda\chi_\Omega$, as $\lambda \to \infty$, has been thoroughly studied (cf., e. g., Herbst and Zhao [HZh], Arendt and Batty [AB]), it seems that little – if anything at all – is known about the corresponding situation of magnetic perturbations of the Laplacian,

$$H(\lambda\vec{a}) = (-i\nabla - \lambda\vec{a})^2 \ ,$$

as $\lambda \to \infty$, where $\vec{a}$ is a vector potential on $\mathbf{R}^n$ of class C^1 or C^2. Recalling that, for Ω an open subset of $\mathbf{R}^n$, the strong resolvent limit of $-\Delta + \lambda\chi_\Omega$ is given by $-\Delta_M$,

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