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Mass Generation in the Large N Gross–Neveu-Model

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Abstract: We study the infrared behaviour of the Euclidean Gross–Neveu-Model with discrete chiral symmetry. Imposing a suitable UV-cutoff we prove that for a large (but finite!) number of fermion components the model has (at least) two pure phases, realized by suitable boundary conditions and that the fermion two-point function decays exponentially.

I. Introduction

We want to study the infrared behaviour of the two-dimensional Euclidean Gross–Neveu model [1] which is formally given through the Lagrangian

$$\mathscr{L} = \overline{\psi}(x)\, i\partial\!\!\!/\, \psi(x) - \frac{\lambda}{2N}(\overline{\psi}(x)\, \psi(x))^2 \tag{1}$$

for $N \gg 1$. Here N is the number of fermion flavours, i.e.

$$\psi = (\psi_1, \ldots, \psi_N)^T, (\overline{\psi} = \overline{\psi}_1, \ldots, \overline{\psi}_N)\,.$$

The coupling λ is supposed to be a constant of order 1. Since we will study the model with an UV-cutoff, it does not make much sense to choose $\lambda \gtrsim \pi$: we will show that the model is massive and that the mass approaches the size of the cutoff for large $\lambda > \pi$. If $\lambda \ll \pi$ there arises a technical difficulty: the mass decreases as $e^{-\pi/\lambda}$, and the correlations decay more and more slowly. But the cluster expansions can only be shown to converge for $N \gg m^{-1}$. So small λ requires (very) large N.

Our aim is to show that the mechanism of mass generation discovered by Gross and Neveu and analyzed by them to first order in $1/N$ persists in the full model for N sufficiently large. Being mainly interested in the IR behaviour of the model we will therefore study the model with an UV cutoff, the scale of which is put equal to one.

The UV limit will be postponed to another paper, and one should note in this respect that the UV limit of the two-dimensional massive (by hand!) [2, 3] and of the three-dimensional large N [4] Four-Fermion-Models have already been constructed.