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Generalized Motion by Mean Curvature as a Macroscopic Limit of Stochastic Ising Models with Long Range Interactions and Glauber Dynamics

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Abstract. We study the *macroscopic limit* of an appropriately rescaled *stochastic Ising model* with *long range interactions* evolving with *Glauber dynamics* as well as the corresponding *mean field equation*, which is nonlinear and nonlocal. In the limit we obtain an interface evolving with normal velocity $\theta\kappa$, where κ is *the mean curvature* and the *transport coefficient* θ is identified by an *effective Green–Kubo type formula*. The above assertions are valid for all positive times, the motion of the interface being interpreted in the *viscosity sense* after the onset of the geometric singularities.

1. Introduction

Stochastic Ising models with long range interactions were introduced by Kač, Uhlenbeck and Hemmer in [KUH] (see also Lebowitz and Penrose [LP]) to justify the validity of the Van der Waal's phase diagram, as the interaction range γ^{-1} tends to infinity. For a very comprehensive description of the equilibrium theory of systems with long-range potentials we refer to the paper by Hemmer and Lebowitz [HL].

Stochastic Ising models with Kač potentials evolving in time with *Glauber dynamics* – each spin undergoes in a random way a finite number of flips – have a surprisingly rich structure and exhibit a great variety of physically interesting effects like *spinodal decomposition, development of interfaces, etc.* We refer to the papers by De Masi, Orlandi, Presutti and Triolo [DOPT 1, 2, 3] for a systematic study of some of these properties as well as to the ones by Comets [C], Comets and Eisele [CE] and Lebowitz, Orlandi and Presutti [LOP] for other non-equilibrium properties for systems with Kač potentials.

The *mesoscopic limit* of the *ferromagnetic* stochastic Ising model evolving with Glauber dynamics, i.e. the behavior of the model as $\gamma \to 0$ when space is rescaled by γ and time is kept fixed, was studied by De Masi, Orlandi, Presutti and Triolo

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