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Statistical Properties of Shocks in Burgers Turbulence, II: Tail Probabilities for Velocities, Shock-Strengths and Rarefaction Intervals

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Abstract: This paper studies the structure of the random "sawtooth" profile corresponding to the solution of the inviscid Burgers equation with white-noise initial data. This function consists of a countable sequence of rarefaction waves separated by shocks. We are concerned here with calculating the probabilities of rare events associated with the occurrence of very large values of the normalized velocity, shock-strength and rarefaction intervals. We find that these quantities have tail probabilities of the form $\exp\{-Cx^3\}$, $x \gg 1$. This "cubic exponential" decay of probabilities was conjectured in the companion paper [1]. The calculations are done using a representation of the shock-strength and length of rarefaction intervals in terms of the statistics of certain conditional diffusion processes.

1. Introduction

This paper is a sequel to a previous article [1] concerning the structure of statistical solutions of Burgers' equation with random initial data (Burgers Turbulence)

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{u(x,t)^2}{2}\right) = 0 \\ u(x,t=0) = u_0(x) = \text{ Gaussian white noise}. \end{cases} \tag{1}$$

This nonlinear wave equation is understood as the inviscid limit ($\nu \to 0$) of the equation

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u(x,t)^2}{2}\right) = \nu \frac{\partial^2 u(x,t)}{\partial x^2}. \tag{2}$$

The statistical properties of Burgers Turbulence have been widely studied since the original works of Burgers [2] and Hopf [3]. In particular, expectation values, moments and correlation functions of the velocity, $u(x,t)$, have been calculated. A modern and interesting account of the theory of Burgers Turbulence and other related systems, as well as of its role in recent cosmological theories, can be found in Gurbatov, Malakhoff and Saichev [4]; see also [12].