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On Diffraction by Aperiodic Structures

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Abstract. This paper gives a rigorous treatment of some aspects of diffraction by aperiodic structures such as quasicrystals. It analyses diffraction in the limit of the infinite system, through an appropriately defined autocorrelation. The main results are a justification of the standard way of calculating the diffraction spectrum of tilings obtained by the projection method and a proof of a variation on a conjecture by Bombieri and Taylor.

1. Introduction

Diffraction by aperiodic structures has attracted a lot of attention since the discovery of quasicrystals [50] (for references, see Sect. 6). This paper gives a rigorous treatment of diffraction by aperiodic structures. A brief discussion of quasicrystals is necessary before it is possible to state the results.

Quasicrystals are alloys having long-range order without being periodic. These properties are inferred from diffraction experiments. Their diffraction spectrum consists of bright spots (the "Bragg peaks"), which means that their structure has long-range order (or, for short, "is ordered"). On the other hand the diffraction spectrum has a symmetry that cannot occur in three-dimensional periodic structures (the symmetry is "crystallographically forbidden"). The structure of quasicrystals, therefore, is not periodic. This is where quasicrystals differ from crystals: the diffraction spectrum of crystals also consists of bright spots but their structure is periodic.

The structure of quasicrystals can, in first approximation, be modelled by aperiodic tilings like those obtained by the so-called projection method (see Sect. 5), in the following way. Let X be the set of vertices of a tiling generated by the projection method and consider

$$\mu := \sum_{x \in X} \delta_x , \tag{1.1}$$

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