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Minor Identities for Quasi-Determinants and Quantum Determinants

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Abstract: We present several identities involving quasi-minors of noncommutative generic matrices. These identities are specialized to quantum matrices, yielding q-analogues of various classical determinantal formulas.

1. Introduction

A common feature of the algebraic constructions which originated from the quantum inverse scattering method is the systematic use of matrices T with noncommutative entries, obeying a relation of the form

$$RT_1T_2 = T_2T_1R \,,$$

where the R-matrix is a solution of the Yang–Baxter equation [13, 20, 35]. The entries of the monodromy matrix T may be regarded as the generators of an associative algebra subject to the above relation. Many interesting examples of algebras arise in this way. Among them are $A_q(GL_n)$, the quantized algebra of functions on GL_n [35], the quantized universal enveloping algebra $U_q(gl_n)$ [13, 20, 35], the Yangian $Y(gl_n)$ [13, 33, 27] and the quantized Yangian $Y_q(gl_n)$ [8]. In each of these cases, an appropriate concept of *quantum determinant* can be defined [22, 21, 35] which is of fundamental importance in the description of the center of these algebras and their representation theory. For example the Drinfeld generators [14] of the Yangian $Y(gl_n)$ are given by some *quantum minors* of the T-matrix. These generators can be used to construct the Gelfand–Zetlin bases for certain irreducible representations of $Y(gl_n)$ [30, 26]. Moreover, it is shown in [30] that the Gelfand–Zetlin formulas for $U_q(gl_n)$ follow from certain algebraic identities satisfied by quantum minors of the T-matrix corresponding to the quantized Yangian $Y_q(gl_n)$. Another application of quantum determinants is the construction of a q-deformation of the coordinate ring of the Grassmannian and the flag manifold, whose basis consists in products of quantum minors of the T-matrix associated with the algebra $A_q[GL_n]$ [23, 38]. In this case, the quadratic relations satisfied by quantum minors can be used to establish an analogue of the classical straightening formula [6]. These examples