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# Superspace Formulation of the Chern Character of a Theta-Summable Fredholm Module

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**Abstract:** We apply the concepts of superanalysis to present an intrinsically supersymmetric formulation of the Chern character in entire cyclic cohomology. We show that the cocycle condition is closely related to the invariance under supertranslations. Using the formalism of superfields, we find a path integral representation of the index of the generalized Dirac operator.

## I. Introduction

*I.A.* The purpose of this note is to present an intrinsically supergeometric formulation of the Chern character of [7] in entire cyclic cohomology [3]. The concept of a Fredholm module is closely related to the structure of supersymmetric quantum theory. The construction of the Chern character associated with a $\Theta$-summable Fredholm module presented in [7] uses in an essential way ideas adopted from supersymmetric quantum field theory. The $n^{\text{th}}$ component of the Chern character is written as a certain finite temperature $(n+1)$-point Schwinger function integrated over an $n$-simplex. The physical interpretation of the closedness of the Chern character under Connes' coboundary operator $\partial$ remained, however, unclear.

The construction of this paper is based on the simple observation that a more natural form of the Chern character arises if the integrals over simplexes are replaced by Berezin integrals over supersimplexes. (A supersimplex is a superdomain whose base is an ordinary simplex.) This makes the supersymmetric nature of the Chern character transparent. We find that the Chern character is invariant under the $(1|1)$-dimensional supergroup of translations in the time direction and in one extra fermionic direction. It is the invariance under supertranslations in the fermionic direction which is equivalent to the closedness of the Chern character under $\partial$. Furthermore, this form of the Chern character lends itself well to a path integral representation. This should be a useful technical tool in studying the topological properties of Fredholm modules arising in quantum field theory. Our discussion

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