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Representation of the Category of Tangles by Kontsevich's Iterated Integral

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Abstract: Applying Kontsevich's iterated integral for tangles, we get an isotopy invariant of tangles. We give a method to compute the integral of a tangle combinatorially from modified integrals of some simple tangles. We localize the integral by moving the end points of the tangle to an extreme configuration, and modify the integral so that it is convergent. By using a similar technique, we generalize Kontsevich's invariant to a framed tangle.

Introduction

After the Jones polynomial was discovered, many invariants of links are constructed. Almost all of them are coming from solutions of the quantum Yang-Baxter equation. On the other hand, Vassiliev [25] constructed a wide family of knot invariants. Let χ_h be a knot invariant coming from a solution $R(h)$ of the Yang-Baxter equation with a parameter h such that $R(0)$ is the trivial solution. Then, $d^k\chi_h/dh^k|_{h=0}$ is contained in Vassiliev's family of invariants. Hence, Vassiliev's invariants include many invariants, e.g. the Alexander, Jones, Homfly, Kauffman polynomials and their generalizations in [1, 17, 20, 22], etc. Kontsevich gives a universal construction of Vassiliev's invariant by using an "algebra of chord diagrams" and "iterated integrals." Let V_k denote the space of Vassilev's invariants of degree less than $k+1$. By studying combinatorial properties of invariants in V_k, he constructs a module $\mathscr{A}_0^{(k)}$ spanned by chord diagrams on a circle with relations in Fig. 1 which correspond to the combinatorial relations for Vassiliev's invariants given in [7]. He shows that $V_k/V_{k-1} = (\mathscr{A}_0^{(k)})^*$, the dual space of $\mathscr{A}_0^{(k)}$. Let $\mathscr{A}'_0 = \bigoplus_{k=0}^{\infty} A_0^{(k)}$. Then $\mathscr{A}'_0$ has a graded algebra structure with a product coming from the connected sum of chord diagrams. The 4-term relation assures the well-definedness of the above product, i.e. the product does not depend on the positions of the strings we cut to produce the connected sum. Let $\mathscr{A}_0$ denote the formal completion of $\mathscr{A}'_0$ with

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