Generalized Hypergeometric Functions and Rational Curves on Calabi–Yau Complete Intersections in Toric Varieties

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Abstract. We formulate general conjectures about the relationship between the A-model connection on the cohomology of a d-dimensional Calabi–Yau complete intersection V of r hypersurfaces $V_1, \ldots, V_r$ in a toric variety $\mathbf{P}_\Sigma$ and the system of differential operators annihilating the special generalized hypergeometric series Φ_0 constructed from the fan Σ. Using this generalized hypergeometric series, we propose conjectural mirrors V' of V and the canonical q-coordinates on the moduli spaces of Calabi–Yau manifolds.

In the second part of the paper we consider some examples of Calabi–Yau 3-folds having Picard number > 1 in products of projective spaces. For conjectural mirrors, using the recurrent relation among coefficients of the restriction of the hypergeometric function Φ_0 on a special line in the moduli space, we determine the Picard–Fuchs equation satisfied by periods of this special one-parameter subfamily. This allows to obtain some sequences of integers which can be conjecturally interpreted in terms of Gromov–Witten invariants. Using standard techniques from enumerative geometry, first terms of these sequence of integers are checked to coincide with numbers of rational curves on Calabi–Yau 3-folds.

1. Introduction

In this paper we consider complex projective smooth algebraic varieties V of dimension d whose canonical bundles $\mathscr{K}_V$ are trivial, i.e. $\mathscr{K}_V \cong \mathscr{O}_V$, and the Hodge numbers $h^{p,0}(V)$ are zero unless $p = 0$, or $p = d$. These varieties are called d-*dimensional Calabi–Yau varieties*, or *Calabi–Yau d-folds*. For each dimension $d \geqq 3$, there are many examples of topologically different Calabi–Yau d-folds which can be constructed from hypersurfaces and complete intersections in weighted projective spaces [5, 6, 7, 24, 23, 26].

Physicists have discovered a fascinating phenomen for Calabi–Yau manifolds, so-called *mirror symmetry* [12, 17, 27, 29]. Using mirror symmetry, Candelas et al. in [9] have computed the coefficients of the q-expansion of the Yukawa coupling for Calabi–Yau hypersurfaces of degree 5 in $\mathbf{P}^4$. The method of Candelas et al. was applied to Calabi–Yau 3-folds in weighted projective spaces [14, 33, 21] and