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A Regularity Theorem for Solutions of the Spherically Symmetric Vlasov–Einstein System

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Abstract: We show that if a solution of the spherically symmetric Vlasov–Einstein system develops a singularity at all then the first singularity has to appear at the center of symmetry. The main tool is an estimate which shows that a solution is global if all the matter remains away from the center of symmetry.

1. Introduction

This paper is concerned with the long-time behaviour of solutions of the spherically symmetric Vlasov–Einstein system. In [4] a continuation criterion was obtained for solutions of this system, and it was shown that for small initial data the corresponding solution exists globally in time. In the following we investigate what happens for large initial data. In the coordinates used in [4] the equations to be solved are as follows:

$$\partial_t f + e^{\mu-\lambda}\frac{v}{\sqrt{1+v^2}}\cdot\partial_x f - \left(\frac{x\cdot v}{r}\dot{\lambda} + e^{\mu-\lambda}\sqrt{1+v^2}\mu'\right)\frac{x}{r}\cdot\partial_v f = 0\,, \tag{1.1}$$

$$e^{-2\lambda}(2r\lambda' - 1) + 1 = 8\pi r^2\rho\,, \tag{1.2}$$

$$e^{-2\lambda}(2r\mu' + 1) - 1 = 8\pi r^2 p\,, \tag{1.3}$$

$$\rho(t,x) = \int\sqrt{1+v^2}f(t,x,v)dv\,, \tag{1.4}$$

$$p(t,x) = \int\left(\frac{x\cdot v}{r}\right)^2 f(t,x,v)\frac{dv}{\sqrt{1+v^2}}\,. \tag{1.5}$$

Here x and v belong to $\mathbb{R}^3$, $r := |x|$, $x\cdot v$ denotes the usual inner product of vectors in $\mathbb{R}^3$, and $v^2 := v\cdot v$. The distribution function f is assumed to be invariant under simultaneous rotations of x and v, hence ρ and p can be regarded as functions of t and r. Spherically symmetric functions of t and x will be identified with functions of t and r whenever it is convenient. In particular λ and μ are regarded as functions of t and r, and the dot and prime denote derivatives with respect to t and r respectively. It is assumed that $f(t)$ has compact support for each fixed t. We