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Yang–Mills and Dirac Fields in a Bag, Existence and Uniqueness Theorems

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Abstract: The Cauchy problem for the Yang–Mills–Dirac system with minimal coupling is studied under the MIT quark bag boundary conditions. An existence and uniqueness theorem for the free Dirac equation is proven under that boundary condition. The existence and uniqueness of the classical time evolution of the Yang–Mills–Dirac system in a bag is shown. To ensure sufficient differentiability of the fields we need additional boundary conditions. In the proof we use the Hodge decomposition of Yang–Mills fields and the theory of non-linear semigroups.

1. Introduction

The present paper is part if a series devoted to the study of the classical theory of Yang–Mills fields. Its aim is to establish the existence and uniqueness theorem for Yang–Mills–Dirac fields satisfying modified bag boundary conditions on a contractible bounded domain $M \subset \mathbb{R}^3$. Since the domain M is fixed, our result corresponds to a static bag with zero tension. In Minkowski space the classical Yang–Mills equations have been studied in refs. [1–3]. The existence and uniqueness result for the pure Yang–Mills theory under bag boundary conditions was obtained in [4]. Here, we extend that result to include minimal interaction between the Yang–Mills field and the Dirac field.

Since classical non-abelian Yang–Mills fields are not observed in nature, one may argue that the classical Yang–Mills theory is not relevant to physics. However, the understanding of many physical phenomena in gauge theory, like conservation laws for colour charges, are based on the classical notions for the Yang–Mills theory. It is the knowledge of the classical structure of the theory, together with an appropriate understanding of the process of quantization, which enables one to arrive at a proper description of possible quantum phenomena.

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