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About the Regularizing Properties of the Non-cut-off Kac Equation

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Abstract: We prove in this work that under suitable assumptions, the solution of the spatially homogeneous non-cut-off Kac equation (or of the spatially homogeneous non cut-off 2D Boltzmann equation with Maxwellian molecules in the radial case) becomes very regular with respect to the velocity variable as soon as the time is strictly positive.

1. Introduction

In the upper atmosphere, a gas is described by the nonnegative density $f(t,x,v)$ of particles which at time t and point x, move with velocity v. Such a density satisfies the Boltzmann equation (cf. [Ce], [Ch, Co], [Tr, Mu]):

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f = Q(f)\,, \tag{1.1}$$

where Q is a quadratic collision kernel acting only on the variable v and taking in account any collisions preserving momentum and kinetic energy:

$$Q(f)(v) = \int_{v_* \in \mathbb{R}^3} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \{f(v')f(v'_*) - f(v)f(v_*)\} B(|v - v_*|, \theta) \sin\theta \, d\phi d\theta dv_*\,, \tag{1.2}$$

with

$$v' = \frac{v + v_*}{2} + \frac{|v - v_*|}{2}\sigma\,, \tag{1.3}$$

$$v'_* = \frac{v + v_*}{2} - \frac{|v - v_*|}{2}\sigma\,, \tag{1.4}$$

$$\cos\theta = \sigma \cdot \frac{v - v_*}{|v - v_*|}\,, \tag{1.5}$$