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Mapping Class Group Actions on Quantum Doubles

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Abstract: We study representations of the mapping class group of the punctured torus on the double of a finite dimensional possibly non-semisimple Hopf algebra that arise in the construction of universal, extended topological field theories. We discuss how for doubles the degeneracy problem of TQFT's is circumvented. We find compact formulae for the $S^{\pm 1}$-matrices using the canonical, non-degenerate forms of Hopf algebras and the bicrossed structure of doubles rather than monodromy matrices. A rigorous proof of the modular relations and the computation of the projective phases is supplied using Radford's relations between the canonical forms and the moduli of integrals. We analyze the projective $SL(2, Z)$-action on the center of $U_q(sl_2)$ for q an $l = 2m + 1^{\text{st}}$ root of unity. It appears that the $3m + 1$-dimensional representation decomposes into an $m + 1$-dimensional finite representation and a $2m$-dimensional, irreducible representation. The latter is the tensor product of the two dimensional, standard representation of $SL(2, Z)$ and the finite, m-dimensional representation, obtained from the truncated TQFT of the semisimplified representation category of $U_q(sl_2)$.

1. Introduction

Since the seminal paper of Atiyah [A] on the abstract definition of a topological quantum field theory (TQFT) much progress has been made in finding non-trivial examples and extended structures. The most interesting developments took place in three dimensions where actual models of quantum field theory, like rational conformal field theories and Chern–Simons theory led to the discovery of new invariants. See [Cr] and [Wi].

In an attempt to counterpart these heuristic theories by mathematically rigorous constructions the field theoretical machinery had been replaced by quasitriangular Hopf algebras, or quantum groups. The resulting invariants are described in [TV] and [RT]. From here it is not hard to understand how to associate a TQFT to a rigid, abelian, monoidal category and an extended TQFT to a braided tensor category (BTC). In order for these theories to be well defined one has to make a few